A SPACETIME DISCONTINUOUS GALERKIN METHOD FOR WAVE PROPAGATION AND SCATTERING IN SOLIDS[1]

R. Abedi[a], **Y. Fan**[a], **M. Hawker**[a], **L. Yin**[a] and **R. B. Haber**[a]

[a]Department of Theoretical and Applied Mechanics
Center for Process Simulation & Design
University of Illinois at Urbana–Champaign
Urbana, IL 61801, USA
{rabedi, yongfan, hawker, linyin, r-haber}@uiuc.edu

We present a spacetime discontinuous Galerkin (SDG) finite element method for elastic wave propagation that is especially effective for problems involving shocks [1, 2]. We use differential calculus on manifolds to develop a new spacetime formulation for linearized elastodynamics that begins with an integral statement of momentum balance over an arbitrary control volume in the spacetime analysis domain Ω. This leads to an equivalent localized system which includes the jump conditions that govern discontinuous response across shocks and material interfaces.

A straightforward Bubnov-Galerkin weighted residual procedure generates a weak problem statement for a solution with displacement derivatives in $\mathrm{BV}(\Omega)$. We obtain the SDG finite element method by restricting the solution to a finite-dimensional subspace defined with respect to an element partition of Ω. Despite its simple Bubnov-Galerkin format, the new SDG method satisfies momentum balance exactly on each spacetime element, is high-order stable and exhibits optimal convergence in smooth regions. There are no extra stabilization terms and no tuning parameters. A direct element-by-element solution scheme, with $\mathcal{O}(N)$ computational complexity and a rich parallel structure, can be used when the spacetime mesh satisfies a certain causality cone constraint.

In this presentation, we emphasize applications of the SDG finite element method to problems involving elastic wave propagation and scattering in elastic bodies. In particular, we consider wave propagation due to impact loading and the problem of crack-tip wave scattering. The SDG solutions are remarkably free of numerical artifacts, and there is excellent agreement in the wave-scattering problem between our numerical predictions and analytical solutions for the history of the dynamic stress intensity factor.

References

[1] L. Yin, A. Acharya, N. Sobh, R. Haber, D. Tortorelli, "A space–time discontinuous Galerkin method for elastodynamic analysis," in B. Cockburn, G. E. Karniadakis, and C. -W. Shu, eds., *Discontinuous Galerkin Methods: Theory, Computation and Applications*, pp. 459-464, Springer, (2000).

[2] L. Yin, "A New Spacetime Discontinuous Galerkin Finite Element Method for Elastodynamic Analysis," Thesis submitted in partial fulfillment of the requirements of the degree of Doctor of Philosophy in Theoretical and Applied Mechanics, Graduate College, University of Illinois at Urbana-Champaign, 2002.

[1]Supported by NSF grants DMS 98-73945 and DMR 01-21695 and DOE subcontract B341494.